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Article

Regimes and Critical Thresholds of Insurance Companies' Financial Stability

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Abstract: This study is devoted to identifying the nonlinear regularities governing the stability of insurers in Uzbekistan. The dynamics of the Financial Stability Index (FSI) were analysed using B. Hansen's threshold panel regression, based on quarterly data for 34 companies for 2022–2026. The threshold by company size (annual premiums of approximately UZS 110 billion at 2025 prices, or approximately 2% of market share) is highly significant. It is preserved across nominal, real and relative size measures, in non-overlapping annual data, and in a version of the index without the size component. The threshold for the reinsurance share (8.6%) is also significant, but its stability is low. A "stability trap" based on the index's own level was not confirmed. The regimes differ not through inertia, but through the conditional equilibrium level under average conditions (61 points for small companies and 81 points for large companies). When company effects are included, this difference weakens, meaning that it mainly reflects differences between companies. On the stability map constructed in the size and reinsurance plane, only one of the four regimes has its conditional equilibrium in the low zone. At the same time, the interaction of these two factors was not statistically confirmed. The stochastic cusp model has an advantage according to the information criterion, but no bistability dynamics were identified during the observed period.

Keywords: insurance company, financial stability, threshold regression, conditional equilibrium, reinsurance, stability map, catastrophe theory, stability tests.

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1. Introduction

Over the last five years, the insurance market of Uzbekistan developed rapidly. During 2020–2025, the volume of collected insurance premiums increased from UZS 1.88 trillion to UZS 12.88 trillion. At the same time, the number of companies operating in the market decreased. The number of companies that entered or left the market exceeded fifteen. In Resolution No. PP-108 of the President of the Republic of Uzbekistan dated 1 March 2024, improving the stability of insurers' activities was identified as one of the main priorities [1]. To carry out this task, the supervisory authority needs to assess the condition of companies correctly and see in advance where risk is accumulating.

It is observed that most econometric models used to assess the stability of insurance companies have a linear structure. In them, it is assumed that the effect of a factor and the speed at which an indicator returns to its own average are the same for all companies in the market. This assumption is convenient, but it contradicts an important observation in insurance practice. When a single large loss occurs, the condition of small and large companies changes differently. There is also a difference in the dynamics of a company

with a diversified portfolio that cedes part of its risk to reinsurance and a company that retains all its risk.

Three forms of nonlinearity are of particular importance in financial stability theory. The first is dependence on regimes, that is, the law governing changes in a company's stability (the equilibrium level and the speed of return to it). This differs depending on which group the company belongs to, for example, whether it is small or large. The second is jumps. In this case, even if conditions change slowly, the company's condition moves at once to another level. In mathematics, such phenomena are studied by catastrophe theory, founded by R. Thom and E. Zeeman [2]; [3]. According to this theory, under a certain combination of control factors, a system has two stable states. In such a region, a small change in a factor causes the system to jump from one state to another. When the factor returns to its previous value, the system does not immediately return. The conclusion from this is that a company operating stably may suddenly fall into a crisis state when conditions gradually worsen. To emerge from such a state, it is not enough simply to return conditions to their previous level. The issue of modelling the financial stability of an insurance company on the basis of catastrophe theory was considered in the author's 2014 monograph through impulse processes in signed directed graphs [4]. In this article, the idea from that monograph is statistically tested using empirical data. The third is critical points. In this case, the relationship between an external factor and the insurance result changes its direction or strength at a certain value of that factor. For example, when inflation is low, the loss ratio may decrease due to growth in nominal premiums. But when inflation exceeds a certain threshold, claims begin to become more expensive faster than premiums, and its effect on insurance results changes to the opposite direction. Critical points and dependence on regimes differ from each other. This difference is related not to the company's own characteristics, but to macroeconomic conditions that affect all companies at the same time. The third form, that is, the critical level of inflation, was considered in a separate article by the author [5]. This article examines the first two forms. For the policy of the supervisory authority, these forms lead to different conclusions. When choosing supervisory measures, it is important whether low stability in small companies is a temporary condition or a structural feature.

The aim of the article is to statistically test the first two forms of nonlinearity in the dynamics of the financial stability of insurance companies in Uzbekistan, namely, dependence on regimes and jumps. For this, threshold panel models and the stochastic catastrophe model were used. The reliability of the results was tested in a number of alternative specifications. These consist of real and relative measures of size, company effects, non-overlapping annual data, and a version of the index without the size component. At the end of the study, two practical tools are proposed for the supervisory authority. These are a stability map, and its possibilities and limitations.

The theory of threshold regression models was formed in the works of B. Hansen. He proposed a bootstrap method for testing linearity when a parameter is not identified under the null hypothesis [6]. He also developed a method for estimating the threshold value and constructing its confidence interval [7], as well as a threshold model for panel data [8]. Its essence is that the sample is divided into two based on an unknown value of an observable indicator, and its own coefficients are estimated in each part. This model was applied by M. Khan and A. Senhadji in macroeconomics to determine at what level of inflation it starts to harm economic growth [9]. The works of R. Ibarra and D. Trupkin showed that this threshold in developing countries depends on the quality of institutions [10].

The importance of an insurance company's size in its activity was studied by P. Hardwick and M. Adams in the UK life insurance market [11]. D. Mayers and C. Smith linked the theoretical foundations of the demand for reinsurance with company size, financial condition and risk structure [12]. According to their theory, small and financially

weak companies should use reinsurance more. J. Garven and J. Lamm-Tennant empirically tested these relationships [13]. In Uzbekistan, the mechanisms for transferring risks through reinsurance were studied in the works of T. M. Baymuratov [14] and R. S. Azimov [15].

Another area of nonlinearity is connected with catastrophe theory, developed by R. Thom [2] and E. Zeeman [3]. This theory shows that even when the factors controlling a system change continuously, its state may change discontinuously. To apply a deterministic model to empirical data, L. Cobb added a random term to it. He proposed finding the parameters using the maximum likelihood method [16]. R. Grasman showed that, when testing a cusp model, it is necessary to compare it with alternative models and determine what proportion of observations falls within the bifurcation set [17]. In economic studies, catastrophe models are often applied incorrectly. As a result, the state variable is regressed on its own levels, producing a tautological result. Therefore, only a stochastic model estimated using the maximum likelihood method was used in this article.

The idea of describing instability by a region in the parameter plane, rather than by threshold values of individual indicators, was developed in the dynamics of viscoelastic systems. In that work, regions of instability were constructed in the plane of load parameters [18]. For an insurance company, this approach was considered on the basis of directed graphs in the author's 2014 monograph [4].

An empirical analysis of the stability of insurance companies in Uzbekistan was presented in the work of F. Ergasheva and K. Nomozova [19]. The nonlinear effect of inflation on insurance demand and market loss ratios was considered in a separate work by the author [5]. However, the division into regimes in the dynamics of the stability of insurance companies in the country and their conditional equilibria have not been econometrically estimated.

2. Materials and Methods

Data and the Stability Index

The empirical base was compiled from reports published quarterly by the National Agency of Perspective Projects of the Republic of Uzbekistan. From these reports, data were obtained by company on premium and payment volumes, premiums ceded to reinsurers, contracts, claims and their settlement, from the first quarter of 2020 to the second quarter of 2026. Annual market-level data for 2011–2025 were used to estimate critical inflation.

Data on the quarterly GDP deflator of the Statistics Committee of the Republic of Uzbekistan were used to express premiums in real terms [20]. The deflator is given relative to the corresponding quarter of the previous year. For this reason, for each quarter the price level was reconstructed by the chain method along that quarter's own series, and the 2025 average was set equal to one. Each quarter's premium was first divided by its own price level, and then summed over four quarters.

The Financial Stability Index (FSI), developed by the author, was used as a measure of stability. However, due to the absence of open data on company capital and insurance reserves, capital adequacy is not included in this index. It describes only the underwriting and operational stability of an insurance company. The index differs from the IMF's Financial Soundness Indicators and stress indices at the financial system level, that is, it is calculated at the company level. The index consists of five indicators (Table 1) and is calculated as follows:

$$FSI_{it} = 100 \cdot \sum_j w_j z_{itj}, \quad z_{itj} = \frac{x_{itj} - L_j}{U_j - L_j} \tag{1}$$

Here, $\tilde{x}_{itj}$ is the j-indicator of company i in period t (winsorised within the interval Lj and Uj); Lj and Uj are the lower and upper limits of the j-indicator, that is, its 2.5th and 97.5th percentiles across the entire panel; for indicators that reduce stability, $1 - z_{itj}$ is used instead of z_{itj}. The weights w_j were determined as the normalised geometric mean of the weights from the principal component and entropy methods. The loss ratio was taken not according to the "the lower, the better" principle, but as a deviation from the market benchmark. This is because companies with very low loss ratios are often found among those that exited the market. Based on the terciles of the index distribution, low (below 64.2 points), medium (64.2–76.8 points) and high (above 76.8 points) stability zones were identified. The zones are not statistically estimated parameters, but a classification rule.

Table 1.

Indicators and Weights of the Financial Stability Index (FSI)

Indicator	Calculation method	Effect on stability	Weight
X_1. Size	Natural logarithm of annual premiums	+	0,268
X_2. Deviation of the loss ratio from the market benchmark	$\lvert LR_{it} - LR_{mt} \rvert$, where LR_{mt} is the aggregate market loss ratio	–	0,174
X_3. Loss ratio variability	Standard deviation of the quarterly loss ratio	–	0,196
X_4. Claim settlement rate	Number of claims paid / number of claims received × 100	+	0,178
X_5. Growth proportionality	$\lvert g_{it} - median(g_t) \rvert$, where g_{it} is the annual premium growth rate	–	0,184

Source: Author's own work. Note: all indicators are calculated for the last four quarters; "+" means that the indicator increases as stability increases, and "–" means that it decreases.

The index was calculated for each quarter from the fourth quarter of 2021 to the second quarter of 2026, based on indicators for the last four quarters, that is, using a rolling window. Flow indicators (premiums, payments, claims) were summed over four quarters, while relative indicators were taken as the ratios of these sums. In practice, such an index produces almost the same result as the annual index and increases the number of observations fourfold. However, because adjacent quarters share three quarters of data, the volume of independent information does not exceed this ratio. For this reason, standard errors and the bootstrap were clustered by company. The results were also checked using non-overlapping annual windows. For the threshold models, the sample consists of 383 observations from 34 companies, from the fourth quarter of 2022 to the second quarter of 2026. Companies that exited the market were retained in the sample; otherwise, the results would have shifted toward the "survivors."

Threshold Panel Model

The two-regime threshold model is written as follows [7; 8]:

$$FSI_{it} = (a_1 + \rho_1 FSI_{i,t-4}) \cdot 1(q_{it} \le \gamma) + (a_2 + \rho_2 FSI_{i,t-4}) \cdot 1(q_{it} > \gamma) + \tau_t + e_{it} \quad (2)$$

Here, q_{it} is the indicator that determines the regime (the threshold variable); γ is the threshold value to be estimated; $1(\cdot)$ is a function equal to one when the condition is satisfied and zero otherwise; τ_t are effects common to all companies in each quarter; ρ_1 and ρ_2 are the inertia coefficients in the regimes. The quarter from one year earlier was taken as the lag. The reason is that, in the rolling window, adjacent quarters share three quarters, and a closer lag gives artificial autocorrelation. In a short panel with a lagged dependent variable, fixed effects cause the bias identified by S. Nickell [21]. Therefore, company effects were not included in the model. It should be noted that model (2) is dynamic, and its dynamics are determined not by the time effects τ_t, but by the term $FSI_{i,t-4}$. B. Hansen's panel model [8], however, was developed for panels without a lagged dependent variable. For this reason, model (2) was estimated using the ordinary least squares method without

company effects. The threshold and its confidence interval were based on the approaches presented in [7], while the linearity test was based on [6]. Some of the permanent differences between companies are accounted for through the regimes. Dynamic threshold panel estimators that fully account for them [22]; [23] are a task for future research. As a robustness check, the model was also estimated with company effects (the approach of B. Hansen [8]). In this case, ρ is biased downward due to Nickell bias, and the conditional equilibrium cannot be calculated, but the existence of a threshold can be tested.

In each regime, the index tends toward its own conditional equilibrium level:

$$FSI_r^* = \frac{a_r + \bar{\tau}}{1 - \rho_r}, \quad r=1,2 \qquad (3)$$

Here, $\bar{\tau}$ is the average of the time effects. ρ_r is the inertia coefficient (the larger it is, the more slowly a shock dies out), while the recovery speed is equal to $1 - \rho_r$. FSI_r^* is the conditional equilibrium when the company remains in that regime and time conditions are average, that is, the level to which the index returns. It does not reflect the company's inevitable long-term condition. If size or the reinsurance share changes, the company moves to another regime. This difference is important for supervision. If the regimes differ only in inertia, a low index in a small company will return to the average over time. If they differ in their conditional equilibrium, the low index will remain as long as the company stays in that regime.

The value that minimises the function $S(\gamma)$ was taken as the estimate of γ. Here, $S(\gamma)$ is the sum of squared model residuals for a given γ. To ensure that enough observations remained in each regime, the search was conducted outside the lower and upper 15 per cent of the threshold variable's distribution. The linearity hypothesis ($a_1 = a_2$ and $\rho_1 = \rho_2$) and the confidence interval for γ were tested using the following statistics:

$$F = \frac{n \cdot (S_0 - S(\gamma))}{S(\gamma)}, \quad LR(\gamma) = \frac{n \cdot (S(\gamma) - S(\gamma))}{S(\gamma)} \qquad (4)$$

Here, S_0 is the sum of squared residuals of the linear model without division into regimes ($FSI_{it} = a + \rho FSI_{i,t-4} + \tau_t + e_{it}$); $S(\hat{\gamma})$ is the sum of squared residuals of the two-regime model (2), estimated at the threshold; n is the sample size. Since γ is not identified under the null hypothesis, the F statistic does not follow a standard distribution [6]. This problem was addressed using a wild cluster bootstrap with 1,999 replications [24]. In each replication, all residuals of one company were multiplied by the same random sign (+1 or −1). The 95 per cent confidence interval for γ consists of values satisfying the condition $LR(\gamma) \leq 7.35$ [7].

Four candidate indicators that could determine the regime were tested. The first was the index itself, which tests the "stability trap" hypothesis. That is, it tests whether the mechanism for returning to the average may stop working for a company that falls below a certain level. The other three were company size, the share of premiums ceded to reinsurance, and activity intensity θ. Activity intensity is defined as the logarithm of the ratio of the number of claims received by a company in the last four quarters to that company's average level over its entire observation period. In the main specification, size was measured as the natural logarithm of real annual premiums at 2025 prices. Nominal premiums and market share were used as alternative measures. Within a quarter, companies are ranked in the same order according to these three measures. They differ only in how they account for price increases over time. Since the size and reinsurance thresholds may act simultaneously, at the next stage a four-regime model formed by the intersection of the two thresholds was estimated. For the conditional equilibria calculated using formula (3), 90 per cent intervals were constructed by resampling companies (1,999 replications). The interaction between size and reinsurance was tested in two ways. First,

using a clustered Wald test of the interaction coefficients in the four-regime model. Second, using a bootstrap confidence interval for the difference in differences of the conditional equilibria $\Delta = (FSI_4^* - FSI_3^*) - (FSI_2^* - FSI_1^*)$.

Stochastic Cusp Model

In the cusp model, the system's state y clusters around the "potential wells" of the following potential function:

$$V(y;\alpha,\beta) = -\alpha y - \beta\frac{1}{2}y^2 + \frac{1}{4}y^4 \quad (5)$$

Here, α and β are control parameters. α tilts the potential to one side, while β determines its shape. If $\beta < 0$, the function has one "well," and the state shifts smoothly as α changes. If β is sufficiently large and positive, two "wells" are formed. In this situation, a small change in α moves the company from one stable state to another. For observations to fall within the bifurcation set, that is, the region where two stable states exist at the same time, the discriminant $\delta = 27\alpha^2 - 4\beta^3$ must be negative. In Cobb's stochastic version, the density of y takes the form $exp(-V)$. The control parameters are expressed as linear combinations of observable factors [16].

The standardised quarterly *FSI* was taken as the state variable (476 observations for 35 companies from the first quarter of 2022 to the second quarter of 2026). Since this model does not require the value from the previous year, its sample is larger than that of the threshold model (383 observations). In that model, the first four quarters of each company are used as lags. The parameter α was expressed through company size and five indicators describing the portfolio. These are the share ceded to reinsurance, average premium per policy, frequency of events, average payment per event, and the ratio of liability to premiums. All factors were taken with a one-quarter lag. The parameter β included factors assumed to be capable of producing two stable states, namely θ and inflation. To assess the model quality objectively, it was compared with three competing models [17]. Two of them are linear regressions, one with normally distributed errors and the other with an asymmetric distribution. The third is a logistic surface model.

3. Results and Discussion

Threshold Regimes

The results of the two-regime model are presented in Table 2 and Figure 1.

Table 2. Results of the Threshold Model of FSI Dynamics

Threshold variable	Threshold [95% interval]	*F* (*p*-value)	Low regime: *n; ρ₁; FSI**	High regime: *n; ρ₂; FSI**
FSI ($t-4$)	65,4 [55,9; 70,3]	24,6 (0,224)	131; 0,25; 61,3	252; 0,80; 76,3
ln real annual premiums ($t-4$)	11,61 [11,57; 11,67]	88,4 (0,001)	162; 0,39; 60,6	221; 0,42; 80,8
Reinsurance share, % ($t-4$)	8,6 [4,5; 10,1]	38,2 (0,007)	180; 0,52; 64,3	203; 0,56; 80,8
Activity intensity θ	−0,12 [−0,64; 0,62]	8,8 (0,898)	180; 0,65; 78,4	203; 0,75; 69,7

Source: Author's calculations. Note: N = 383 (34 companies); size is real annual premiums at 2025 prices; p-values are based on the wild cluster bootstrap (1,999 replications); FSI is the conditional equilibrium.*

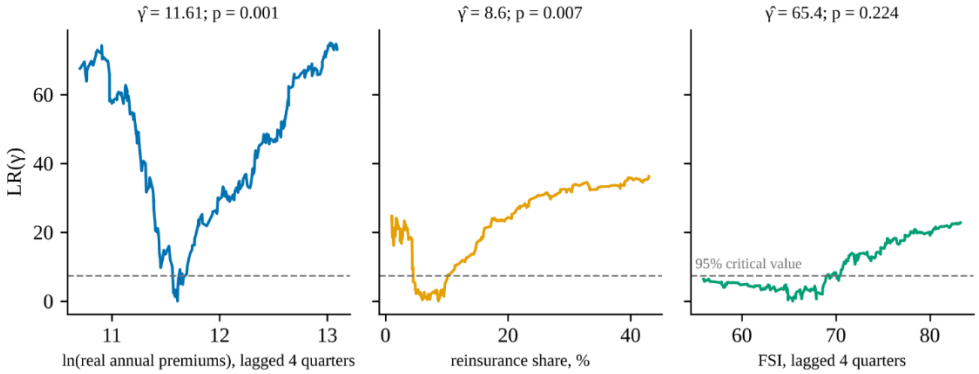

Figure 1. Likelihood Ratio Statistic *LR(γ)* for Threshold Values

Source: Author's calculations. Note: the dashed line is the 95 per cent critical value (7.35).

The "stability trap" hypothesis was not confirmed; that is, the threshold based on the previous level of the index was found to be statistically insignificant ($p = 0.22$). As can be seen in the right-hand part of Figure 1, the *LR(γ)* curve is below the critical value and flat across most of the search interval (approximately 56–70 points), meaning that the threshold value is not precisely identified. Inertia is lower for observations with a low index (0.25 and 0.80), meaning that they return to the average more quickly. Although this difference is significant when considered separately, the linearity hypothesis is not rejected by the bootstrap. Thus, the data do not support the assumption that a company whose index falls loses its ability to recover on its own.

For size and reinsurance, however, linearity is rejected ($p < 0.001$ and $p = 0.007$). Their *LR(γ)* curves have a clear and deep minimum. The size threshold is 11.61 on the logarithmic scale of real premiums, which is equivalent to annual premiums of approximately UZS 110 billion at 2025 prices. Its confidence interval is narrow (UZS 106–116 billion). The reinsurance threshold is 8.6 per cent, with a confidence interval of 4.5–10.1 per cent. The conditional equilibrium in the regime with a low reinsurance share (64.3 points) is located at the boundary between the low and medium zones (64.2), meaning that this indicator alone does not place the equilibrium in the low zone. Since four candidate variables were tested, the effect of multiple testing was also taken into account: after the Bonferroni correction, the size ($p < 0.002$) and reinsurance ($p = 0.026$) thresholds remain significant at the 5 per cent level.

The most important result is how the regimes differ. The inertia coefficients in the two size regimes (0.39 and 0.42) are almost the same; the difference between them (−0.04) is much smaller than its standard error (0.19). The conditional equilibria, however, differ by 20 points, that is, 60.6 and 80.8 points. In other words, the regimes differ not in how quickly a shock dies out, but in the level around which the index takes shape. The conditional equilibrium of small companies is located within the low zone. Even after the shock dies out, as long as the company remains in this regime, its index returns to around the low zone. This can be seen graphically in Figure 2. The point where each regime line intersects the diagonal is its conditional equilibrium.

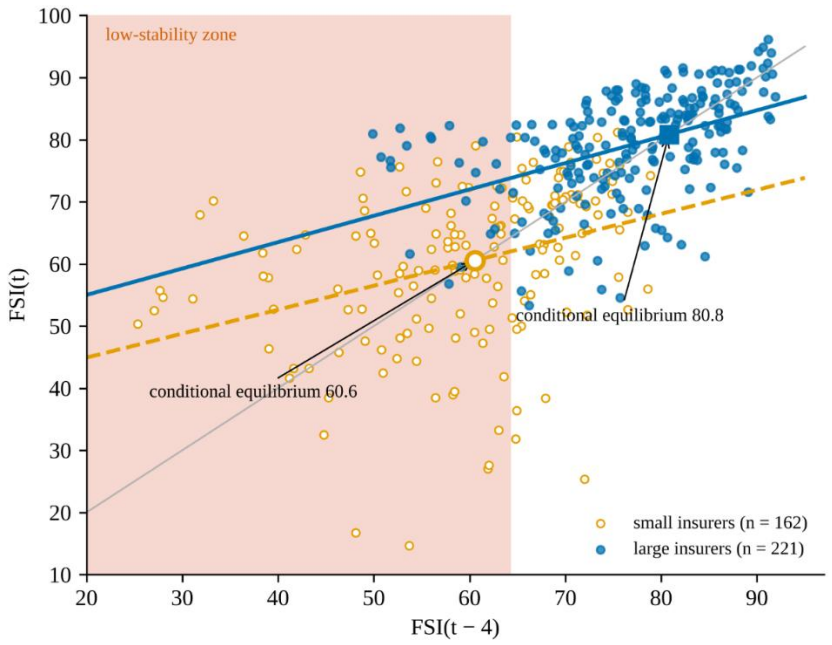

Figure 2. FSI Dynamics and Conditional Equilibrium Points by Size Regime

Source: Author's calculations. Note: "small companies" are companies whose annual premiums four quarters earlier were below UZS 110 billion at 2025 prices; the remaining companies are "large companies"; the yellow dashed line is the small-company regime, the blue solid line is the large-company regime; the thin grey line is FSI(t) = FSI(t–4); the pink area is the low zone (FSI < 64.2); yellow hollow circles are observations for small companies, blue filled circles are observations for large companies; large markers are the conditional equilibrium points.

Since size is an indicator included in the FSI, the possibility that this result is mechanical was specifically checked. The size threshold remained at exactly the same point ($\hat{\gamma}$ = 11.61; p = 0.010) for an index with the size component removed and the weights of the remaining four indicators renormalised. The reinsurance threshold decreased to 5.0 per cent, but remained significant (p = 0.032). Thus, small size leaves a mark not only in the size component of the index, but also in underwriting results, claim payments and growth stability.

No threshold was identified for θ (p = 0.90). The inertia coefficients in the regimes are close (0.65 and 0.75), and the linearity hypothesis is not rejected. In addition, the effect of θ on the index disappears when company size is controlled for. For this reason, this indicator is not interpreted as an independent regime factor.

In conclusion, what distinguishes small and large companies is not inertia, but the conditional equilibrium level.

Robustness of the Results

The reliability of the main result was checked in five ways (Table 3 and Figure 3).

1. Size was measured in three ways: nominal premiums, real premiums at 2025 prices, and market share.
2. The model was estimated using non-overlapping annual windows. For this, only fourth-quarter observations were used. Since the rolling window in the fourth quarter corresponds to the calendar year, these observations are annual FSI, annual premiums and the annual reinsurance share (103 company-year observations).
3. The version of the index without the size component was used.
4. The model was estimated with company effects.
5. The number of bootstrap replications was increased to 1,999.

Table 3.

Robustness Checks for the Threshold Model Results

Specification	$\hat{y}$	*F*	*p*	*n* (low regime)	*FSI** small / low	*FSI** *FSI** large / high
Size threshold						
Real premiums, 2025 prices (main)	11,61 (≈110 billion)	88,4	0,001	162	60,6	80,8
Nominal premiums	11,32 (≈82 billion)	77,2	< 0,001	167	61,1	81,1
Market share, %	1,9	83,5	< 0,001	187	62,5	81,2
Annual data (*N* = 103), real	11,44 (≈93 billion)	26,3	0,002	41	56,3	80,2
FSI without size component, real	11,61	46,8	0,010	162	69,5*	82,1*
Company effects, real	11,61	31,3	0,067	–	–	–
Company effects, nominal	11,08	18,2	0,334	–	–	–
Reinsurance threshold						
Main specification	8,6	38,2	0,007	180	64,3	80,8
Annual data (*N* = 103)	9,5	8,9	0,112	50	61,7	79,8
FSI without size component	5,0	36,2	0,032	–	69,9*	81,4*
Company effects	1,0	10,5	0,828	–	–	–

Source: Author's calculations. Note: N = 383 (34 companies); for annual data, N = 103; p-values are based on the wild cluster bootstrap (1,999 replications); FSI is the conditional equilibrium; * denotes the scale of the index without the size component, which is not directly comparable with the zones of the main index; in the model with company effects, the conditional equilibrium is not calculated because ρ is negative due to Nickell bias.*

The size threshold remained in four checks. When price increases are taken into account, the result becomes stronger (*F* statistic increases from 77.2 to 88.4); using market share, the threshold was found at approximately 1.9 per cent of the share. In the non-overlapping annual data, despite the number of observations being four times lower, the threshold remained significant ($p = 0.002$), while the conditional equilibria changed very little (56.3 and 80.2 points). Thus, the main result is not a product of artificial correlation generated by the rolling window.

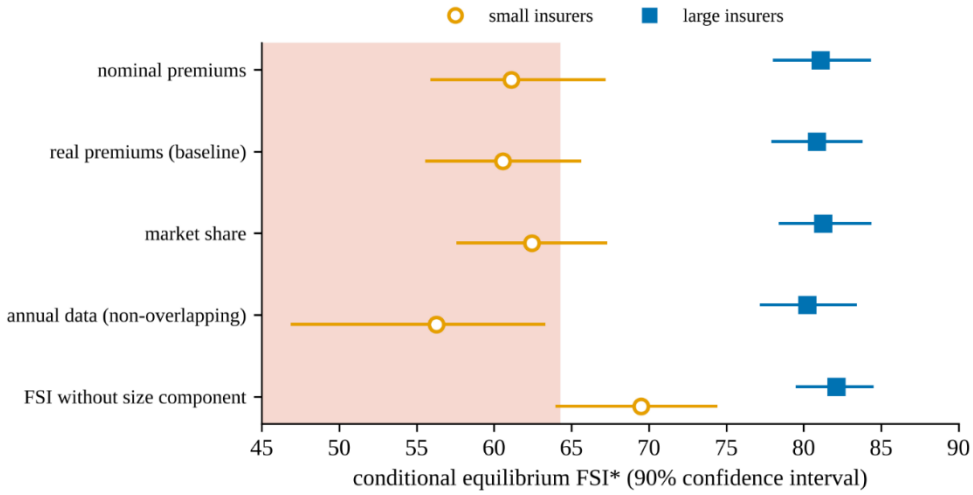

Figure 3. Conditional Equilibria of Small and Large Company Regimes under Different Specifications

Source: Author's calculations. Note: yellow circles indicate the small-company regime and blue squares the large-company regime; lines are 90 per cent confidence intervals based on resampling companies (999 replications); the pink area is the low zone (FSI < 64.2); FSI without the size component is on a different scale.

The model with company effects gives a different picture. In this case, the size threshold is significant only at the 10 per cent level using the real measure ($p = 0.067$), and is insignificant using the nominal measure ($p = 0.33$). This result shows that the difference between regimes is mainly between companies. Differences in size between companies are strongly related to differences in stability, but when a single company grows and crosses the threshold, its index does not jump significantly. This situation can be explained in two ways. First, not small size itself, but characteristics typical of small companies and not observed in the data (business model, management quality, ownership structure) may be associated with lower stability. In addition, it is difficult to identify the within-company effect because few companies crossed the threshold during the short period. For this reason, the article interprets small size not as a cause of low stability, but as an indicator associated with a lower conditional equilibrium.

The reinsurance threshold is less stable. In the index without the size component, it shifts to 5.0 per cent, while in the annual data and in the model with company effects it becomes insignificant ($p = 0.11$ and 0.83). Thus, the size threshold is the main result of the article, while the reinsurance threshold is an additional result that is interpreted with caution.

Stability Map

The two threshold variables are not independent of each other. Spearman's rank correlation is 0.49, meaning that larger companies also have a higher reinsurance share. At first glance, this seems to contradict the theory of D. Mayers and C. Smith [12]. However, according to this theory, demand for reinsurance depends not only on size, but also on portfolio concentration and risk structure. In Uzbekistan, large sums insured for industrial and infrastructure facilities make up a large part of the portfolios of large insurers. Such risks cannot be accepted without reinsurance. Thus, evaluating each threshold separately may mix up their effects. To separate them, a four-regime model was constructed from the intersection of the two thresholds (Table 4).

Table 4.

Conditional Equilibria of FSI by Size and Reinsurance Regimes

Regime	Observations (companies)	Inertia ρ	Conditional equilibrium *FSI**	90% confidence interval	Stability zone
Small, reinsurance ≤ 8.6%	123 (18)	0,37	57,6	[52,8; 63,1]	Low
Small, reinsurance > 8.6%	39 (7)	−0,08	67,3	[64,0; 70,7]	Medium
Large, reinsurance ≤ 8.6%	57 (10)	0,13	75,0	[70,5; 78,1]	Medium
Large, reinsurance > 8.6%	164 (17)	0,47	82,1	[79,0; 85,6]	High

Source: Author's calculations. Note: "small" and "large" mean annual premiums four quarters earlier below and above UZS 110 billion at 2025 prices. Observations are company-quarters from the fourth quarter of 2022 to the second qua rter of 2026 (383 observations, 34 companies); each observation's regime is determined by size and reinsurance share four quarters earlier. Figures in parentheses are the number of unique companies that entered the respective regime at least once during this period (therefore, their sum exceeds 34); confidence intervals are based on resampling companies (1,999 replications).

As can be seen from the table, there is only one regime whose conditional equilibrium falls in the low zone. This is the group of small companies whose reinsurance share does not exceed 8.6 per cent, in which *FSI** is 57.6 points. The upper bound of the confidence interval for this regime (63.1) is also below 64.2 points. When only one of the conditions is met, the conditional equilibrium remains in the medium zone. It is 67.3 points for small companies that use reinsurance, and 75.0 points for large companies with a low reinsurance share. The regime in which neither condition is met falls in the high zone (82.1

points). This regime has few observations and companies (39 observations, 7 companies), so its estimate should be interpreted with caution.

This result does not mean that small size and low reinsurance reinforce each other's effects when they occur together. Each of them separately lowers the conditional equilibrium, and only when both are present does this decrease reach the low zone. The difference in differences of the conditional equilibria is $\Delta = (82.1 - 75.0) - (67.3 - 57.6) = -2.7$ points; its 90 per cent confidence interval [−10.5; 6.8] includes zero ($p = 0.57$). The Wald test of the interaction coefficients also gives only a borderline result ($p = 0.052$). Thus, the effects of size and reinsurance are additive, but their mutually reinforcing effect was not statistically confirmed.

We express this result as a two-dimensional figure (Figure 4). On the map, the area corresponding to the low zone is formed by the intersection of the thresholds along the two coordinates; each threshold considered separately does not define such an area. In conventional supervision, requirements for size and reinsurance are set independently of each other. The map shows that both indicators need to be monitored together. In this sense, the map is an economic analogue of the instability regions constructed for viscoelastic systems [18].

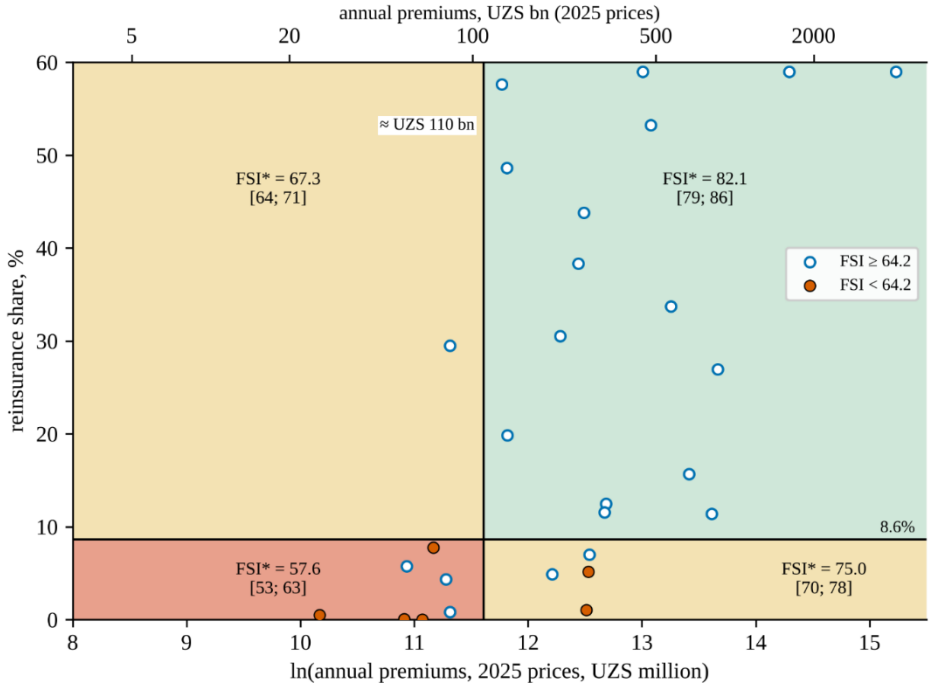

Figure 4. Stability Map in the Plane of Size and Reinsurance Share (2025)

Source: Author's calculations. Note: cells show the conditional equilibrium FSI for each regime and its 90 per cent confidence interval (from Table 4); cell colour indicates the zone in which the conditional equilibrium falls: red – low, yellow – medium, green – high. The lower axis shows the natural logarithm of annual premiums; the upper axis shows annual premiums in UZS billion (both at 2025 prices). Although the model is based on a quarterly panel through the second quarter of 2026, company positions are shown using annual indicators for the last complete calendar year, 2025; premiums of companies that left the market during the year have been annualised. In total, 28 companies operating in 2025 are shown, of which 7 are in the risk area; red dots indicate FSI < 64.2, white dots indicate FSI ≥ 64.2; companies with a reinsurance share above 59 per cent are shown at the upper edge of the graph.*

The map shows the positions of 28 companies as of 2025. Seven of them are in the risk area, and 4 of these already have an index below 64.2 points. Looking at it the other way round, four of the six companies with a low index are located in this area. The 18 companies in Table 4 are those that entered this regime at some point during the entire period, while the seven companies on the map show the position in 2025. As of the second

quarter of 2026 (26 companies, size at 2025 prices), 6 companies are in the risk area, and 3 of the 4 companies with low FSI fall in this area. One of the companies that was in the risk area in 2025 left the market. The map also makes it possible to measure each company's distance to the boundary of the area. For example, a company with annual premiums of approximately UZS 54 billion is located almost twice as far from the boundary along the size coordinate. This distance is a diagnostic indicator; shortening it does not mean that the company's stability will automatically increase.

At the same time, caution is needed when interpreting the map. A low reinsurance share should be considered not as a cause of risk, but as an indicator that requires additional analysis. It may show that a company is retaining large risks, but capital and risk volume were not measured directly. The reinsurance share is an indicator chosen by the company itself. It depends on the portfolio structure, as well as on the price of reinsurance and counterparty risk. Similarly, increasing premiums by itself may increase portfolio concentration and the share of loss-making contracts. Therefore, the map should be considered a diagnostic tool, not a cause-and-effect model. That is, it does not provide a basis for the conclusion that "increasing reinsurance or premiums will raise the index by a certain number of points."

Hypothesis of Abrupt Transitions

Table 5 compares four competing models according to information criteria.

Table 5.

Comparison of the Cusp Model and Alternative Models

Model	Number of parameters	*ln L*	*AIC*	*BIC*
Linear regression (normal error)	10	−400,6	821,3	862,9
Linear regression (asymmetric error)	11	−380,2	782,3	828,2
Logistic surface model	13	−390,6	807,2	861,3
Stochastic cusp model	12	−359,2	742,3	792,3

Source: Author's calculations. Note: N = 476; the state variable is standardised quarterly FSI; AIC and BIC are the Akaike and Bayesian information criteria (the lower, the better).

The cusp model ranks first according to both criteria. By *AIC*, it is better than the linear model by 79.0 units and better than the model with asymmetric errors by 40.0 units. However, this does not yet mean that companies have two stable states. To check this, the estimated α and β values for each observation need to be examined. β is negative for all observations; the 5th and 95th percentiles of its distribution are −4.13 and −3.34, respectively. The discriminant δ is also positive everywhere, with a minimum value of 175. As can be seen in the left-hand part of Figure 5, all points are located far from the bifurcation set. In other words, for each observation, the model implies a single-peaked distribution.

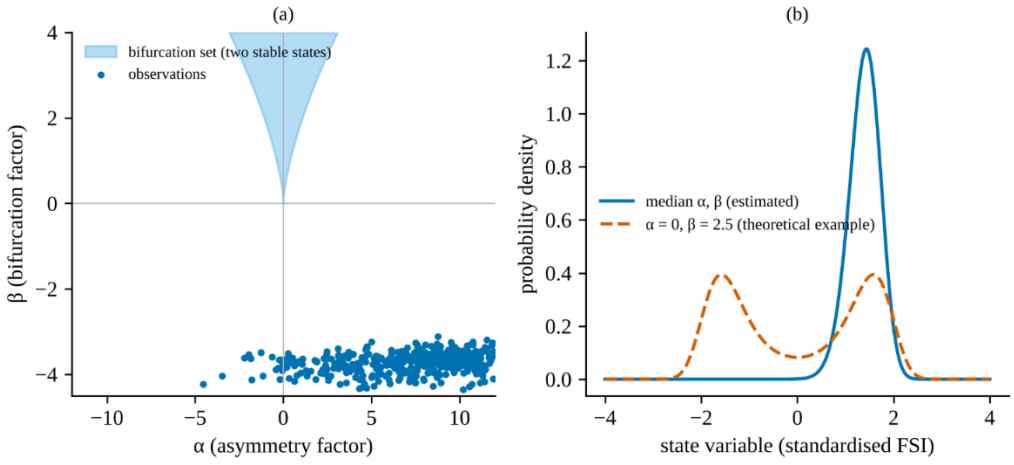

Figure 5. Cusp Model: (a) Location of Observations in the (α, β) Plane and the Bifurcation Set; (b) Estimated and Theoretical Bimodal Distributions

Source: Author's calculations. Note: (a) the light blue area is the bifurcation set (the area in which two stable states exist), blue dots are observations; (b) the blue solid line is the estimated distribution at the median values of α and β, the red dashed line is a theoretical example of a bimodal distribution (α=0, β=2.5).

The advantage of the cusp model according to the information criterion is explained not by bistability, but by the shape of the FSI distribution. The distribution is skewed to the left, with a skewness coefficient of −0.95. This means that for some companies the index falls to very low values. A fourth-degree potential describes this asymmetry better than the normal distribution. The fact that the model with asymmetric errors explains approximately half of the cusp model's advantage also shows this. In the α parameter, company size (3.96), average premium (1.22), event frequency (−0.81) and average payment (−2.13) were significant, while the reinsurance share and liability ratio were insignificant. In the β parameter, the effects of θ and inflation were insignificant (p = 0.27 and 0.20). Thus, during the observed period, inflation did not bring companies closer to the bifurcation region. It should also be noted that standard errors were not clustered by company, so the p-values of coefficients found to be significant may be smaller than their true values. Although FSI is limited to the 0–100 range, in the observations it lies between 14.6 and 96.2 points, with no values close to the boundaries. Therefore, defining the density on the whole real line does not affect the result. The predictive ability of the cusp model was not tested. Here, its purpose is to test bistability.

In conclusion, within the observed period and the selected specification, no "jump" mechanism was identified in insurance companies in Uzbekistan. Nonlinearity in this area exists in another form, namely, as division into regimes and a skewed distribution. The catastrophe hypothesis is not rejected, but the current data do not support it. It can be tested again using data covering longer time series and periods with more severe shocks.

4. Discussion

Comparison of the obtained results with foreign studies reveals several important aspects. The first is connected with the size effect. P. Hardwick and M. Adams studied company size in the UK life insurance market mainly from the point of view of growth rates [11]. Our results show that size plays another role in a developing market. In Uzbekistan, one large contract or one large loss for a small company changes the whole annual result. For a large company, such an event makes up a small part of the portfolio. This makes the size effect in the form of a threshold more logical than a smooth logarithmic relationship.

The second aspect is connected with reinsurance. According to the theory of D. Mayers and C. Smith, demand for reinsurance should be high in small and weak companies [12]. J. Garven and J. Lamm-Tennant checked this theory using data from developed markets [13]. In Uzbekistan, the picture at first glance is the opposite, that is, mainly large companies use reinsurance. The reasons for this can be explained by three factors. These are: large-sum risks are few in the portfolios of small companies; concluding a contract with foreign reinsurers brings additional expenses and rating requirements; and, moreover, the domestic reinsurance market in the country is still at the formation stage. T. M. Baymuratov and R. S. Azimov also noted that risk-transfer mechanisms are not developed sufficiently [14; 15]. This may lead to small companies retaining more risks themselves. For this reason, the positive correlation does not reject the theory, but reflects the connection between portfolio structure and company size in the Uzbek market. At the same time, the reinsurance share in large companies may also be connected with one-off large transactions. For this reason, FSI is sensitive to such transactions too. Since a large transaction accepted and transferred to reinsurance by one of the large companies was not

repeated in 2026, the index fell by almost 20 points in one quarter, although the company's net premiums grew.

The third aspect is connected with the nature of the difference between regimes. In the model with company effects, the size threshold becomes weaker. This means that low stability may be caused not by small size itself, but by other characteristics typical of small companies. For example, business model, management quality or ownership structure. This is also important for supervisory policy. For the supervisory authority, it is more important to identify early and monitor companies in the group with a lower conditional equilibrium than to "move" a company across the threshold. The effect of inflation on the insurance market was considered in a separate article by the author [5]. There, the U-shaped relationship of market loss ratio to inflation and the initial estimate of critical inflation (approximately 10 per cent) are given. In this article, inflation was included only as a bifurcation factor of the cusp model, and its effect was insignificant.

The fourth aspect relates to the practice of applying methods. R. Grasman and co-authors emphasised that the cusp model gives reliable conclusions only when it is compared with alternative models and the share of observations falling within the bifurcation set is shown [17]. Our analysis clearly showed the practical importance of this requirement. If only the information criteria were considered, the advantage of the cusp model could be incorrectly interpreted as proof of catastrophe dynamics. However, the fact that not one observation fell within the bifurcation set rejects this conclusion. In the work of F. Ergasheva and K. Nomozova [19], the stability of insurance companies in Uzbekistan was assessed on the basis of linear models. This article supplements this direction with nonlinear models.

Finally, it is necessary to note the limitations of the study:

- The threshold models were assessed over a short period, from the fourth quarter of 2022 to the second quarter of 2026 (for the one-year lag, data starting from the fourth quarter of 2021 were used).
- Company effects were not included in the main specification, and when they are included, the size threshold becomes weaker.
- Data on company capital, insurance reserves and solvency standards are not public, for this reason the stability index was formed mainly from operational indicators.
- The size indicator of the index was calculated using nominal premiums.
- The size indicator of the index was calculated using nominal premiums, but the effect of price growth on the index level is taken into account in the model through time effects.
- Standard errors of the cusp model were not clustered by company, and its predictive ability was not tested.

Data-related limitations can gradually be removed as new quarters and financial reporting data are added.

5. Conclusion

Based on the conducted study, the following conclusions were reached.

First, the stability of insurance companies in the country does not follow the same regularity for all companies. Regime changes are observed mainly by company size. The threshold corresponds to approximately UZS 110 billion in annual premiums at 2025 prices (approximately 2 per cent of market share). This threshold remains in nominal, real and relative measures of size, in non-overlapping annual data, and in the version of the index without the size component. The threshold by reinsurance share (8.6 per cent) is also significant, but less stable. The threshold based on the index's own level, that is, the "stability trap," was not confirmed.

Second, the regimes differ from each other not by inertia, but by the conditional equilibrium level under average conditions. For small companies this level is 61 points, and for large companies it is 81 points. This shows that the small-company regime is connected with a lower conditional equilibrium. Since this difference weakens when company effects are included, it is not considered a causal effect of small size. It should be interpreted mainly as a difference between companies.

Third, of the four regimes in the size and reinsurance plane, the conditional equilibrium of only one falls in the low zone: the regime in which small size and low reinsurance are observed together (57.6 points). The interaction of the two factors was not statistically confirmed. Their effects add up and lower the equilibrium to the low zone. As of 2025, such companies make up 7 out of 28.

Fourth, although the information criterion gives preference to the stochastic cusp model, there are no observations that fall within the bifurcation set. Thus, within the observed period and specification, in the condition of companies there is nonlinearity in the form of regimes and a skewed distribution, not abrupt transitions.

Based on the study results, the following recommendations are made. First, it is advisable to include the stability map in the supervisory authority's early warning system as an additional analysis layer. It makes it possible to identify early companies in the group with a lower conditional equilibrium. Second, it is not necessary to make reinsurance compulsory for companies in the risk area, because the map is not a causal tool. An in-depth supervisory inspection should be carried out for them. Based on its results, it is advisable to limit the liability retained on the company's own account for one contract by linking it to the amount of capital, or to set an additional capital requirement. Such a requirement would apply not to all companies, but only to those in the risk area. At the same time, it would not make entry into the market more difficult for new participants. Third, size thresholds should be set not in nominal indicators, but in real or relative indicators (for example, market share), and should be reassessed regularly, because a nominal threshold loses its meaning as prices rise.

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