

American Journal of Economics and Business Management

Vol. 9 Issue 11 | **pp.** 644-651 | **ISSN:** 2576-5973

Available online @ https://www.globalresearchnetwork.us/index.php/ajebm

Article

Mathematical Modeling for Evaluating The Efficiency of Public–Private Partnership (PPP) Projects in The Digital Economy

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Abstract: In this study, an integrated mathematical model was developed to assess the effectiveness of public-private partnership (PPP) projects in the digital economy. In the model, the effectiveness function is defined as E = f(I, R, T, S), where I represents investment parameters, R represents digital risk factors, T represents time dynamics, and S represents social outcomes. AHP–TOPSIS integration and Monte Carlo simulation (N = 10,000) were used. The results showed that digital risks reduce project effectiveness by almost 35.2%, and the probability of the project achieving positive effectiveness is 82.0%.

Keywords: Digital Economy, Public–Private Partnership, Mathematical Model, Efficiency Evaluation, Risk Management, AHP–TOPSIS, Monte Carlo Simulation.

Citation: Mustafakulov, S, & Khasan, S. Mathematical Modeling for Evaluating The Efficiency of Public–Private Partnership (PPP) Projects in The Digital Economy.. American Journal of Economics and Business Management 2026, 9(11), 644-651

Received: 13th Aug 2026
Revised: 28th Sep 2026
Accepted: 15th Oct 2026
Published: 17th Nov 2026

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1. Introduction

The rapid development of digital technologies has fundamentally changed the landscape of financing, construction and management of infrastructure projects. Many countries around the world are increasingly turning to public-private partnership (PPP) mechanisms to implement large infrastructure projects that include digital solutions [1, 2]. In this context, the combination of PPP structures with digital economy principles – cloud computing, big data analytics (Big Data), Internet of Things (IoT) and artificial intelligence – has created a new category of projects that are much more complex than traditional infrastructure projects [3, 4].

PPPs in the digital economy differ from traditional partnerships in a number of ways. First, the rapid pace of technological obsolescence poses a unique temporal risk: digital platforms and systems can become obsolete within a few years, while the useful lives of traditional infrastructure assets such as roads or bridges are measured in decades [5]. Second, the intangible nature of many digital outputs – software platforms, data ecosystems, cybersecurity architectures – makes it difficult to quantify the results using standard financial metrics such as net present value (NPV) or internal rate of return (IRR) alone [6, 7]. Third, the interdependence between public information management systems and the innovative capacity of the private sector creates regulatory and institutional uncertainties [8].

However, the existing literature on PPP project evaluation remains largely based on methodologies developed for physical infrastructure. Classical approaches – discounted

cash flow analysis, cost-benefit analysis and value for money (VfM) assessments – assume relatively stable demand forecasts, well-defined output specifications and predictable risk allocations [9]. These assumptions are often violated in digital PPP projects, where user engagement indicators can fluctuate dramatically, project scope evolves iteratively and cybersecurity threats can create non-linear risk dynamics. Therefore, there is a growing recognition of the need for new analytical tools that allow the integration of financial, social, technological and risk dimensions into a single evaluation framework [11].

In recent years, a number of studies have attempted to fill this gap. Multi-criteria decision-making (MCDM) methods, including the analytical hierarchy process (AHP) and the top-order similarity technique (TOPSIS), have been used to select and prioritize PPP projects [12]. Stochastic simulation approaches, particularly Monte Carlo methods, have been used to model uncertainty in project cash flows and risk parameters [13]. Integrative mathematical models combining real options theory and game theory have also been proposed to evaluate PPP projects under uncertainty [14]. However, these contributions tend to view financial performance, social impact, and digital readiness as separate evaluation streams rather than as interrelated components of a single system.

This study aims to fill this gap by developing an integrated mathematical model for assessing the efficiency of PPP projects operating in the digital economy. The proposed model defines the efficiency function as E = f(I, R, T, S), where I represents the investment parameters, R represents the digital risk factors, T represents the time dynamics, and S represents the social outcomes. The rest of the paper is structured as follows: Section 3 describes the materials and methods, Section 4 presents the results, Section 5 discusses the results, and Section 6 draws conclusions.

1.1 Statement of Problem

This study develops an integrated mathematical model to assess the effectiveness of public-private partnership (PPP) projects in the digital economy. The theoretical basis of the model is threefold: (1) financial performance indicators based on discounted cash flow analysis [1]; (2) multi-criteria decision-making (MCDM) methods, in particular AHP and TOPSIS [12]; and (3) Monte Carlo simulation for modeling risk parameters under uncertainty [13]. The research data were collected from secondary sources - the World Bank PPP database, scientific articles indexed in Scopus and Web of Science, and state statistics of the Republic of Uzbekistan.

Group I. Financial performance indicators (I): The following indicators are used to assess the financial viability of the project [6, 7]:

$$NPV = \frac{\Sigma(t = 0..n)\ CFt}{(1 + r)^t} \qquad (1)$$

where CFt is the cash flow in period t, r is the discount rate, n is the project duration (in years). The internal rate of return (IRR) is defined as the value of r* that satisfies the equation NPV = 0.

Group II. Digital risk indicators (R): The risks inherent in digital PPP projects are expressed by the following parameters: R1 – technological obsolescence risk, R2 – cybersecurity risk, R3 – regulatory risk, R4 – integration risk. Integral risk indicator:

$$R = \Sigma(j = 1..4) wj \cdot Rj \qquad (3)$$

where wj is the weight coefficient of the j-risk type (Σwj = 1), determined based on expert assessment using the AHP method [12].

Group III. Time factor indicators (T): To take into account the shortened life cycle of digital projects:

$$T = \alpha \cdot e^{(-\beta \cdot t)} \qquad (4)$$

where α is the initial coefficient of utility of the technology, β is the parameter of the obsolescence rate, t is time (in years) [5].

Group IV. Social outcome indicators (S): Sub-indicators: S1 – increase in the number of users, S2 – reduction in service time, S3 – level of user satisfaction, S4 – job creation.

$$S = \Sigma(k = 1..4)\ vk \cdot \hat{S}k \qquad (5)$$

where vk is the weight coefficient of the k-indicator, Ŝk is the normalized value (from 0 to 1) [9, 11].

The multiplicative-divisional form was chosen to combine the four groups of indicators described above into a single efficiency function. The choice is based on three theoretical considerations.

First, in the literature on multi-criteria decision-making (MCDM), two main families of aggregation are distinguished: compensatory (summary form) and non-compensatory (multiplicative form) [10; 15]. The summative form allows the zero value of one of the components to be fully compensated by the others (for example, the social result may be zero, but the overall efficiency may be high through financial income). This contradicts the essence of PPP projects: public-private partnerships require both financial viability and social utility. The multiplicative form overcomes this weakness: if any component is zero, the overall efficiency also tends to zero.

Second, the time factor (T) is constructed as an exponentially decreasing function (formula 4), which makes its multiplicative position natural: the utility of the technology acts as a discounting coefficient for the components Iˆand Sˆ.

Third, a numerical risk factor (R) is introduced as a denominator in the form (1+R). This approach is consistent with the logic of standard financial discounting: as risk increases, efficiency decreases nonlinearly, but does not become zero (since R∈[0,1] and 1+R∈[1,2]). This is consistent with the discounting approach used by Kováčová et al. (2023) in the evaluation of investment projects [13].

To make the components dimensionless, the financial indicator is normalized:

$$\hat{I} = \frac{NPV}{I_0} \qquad (6)$$

where Iˆis the net present value of the investment (dimensionless). The time factor T is already in the range [0,1] (according to formula 4), the social indicator S is also normalized to [0,1] (according to formula 5), and the risk R is also evaluated on the scale [0,1] (according to formula 3).

Thus, the efficiency function is defined as follows:

$$E = \frac{\hat{I} \cdot T \cdot S}{1 + R} \qquad (7)$$

where E is a dimensionless efficiency index. An index value of E>0 indicates a positive efficiency of the project; the larger E, the more efficient the project. The same normalization procedure must be used to compare different projects.

The Analytical Hierarchy Process (AHP) method is used to determine the weight coefficients in the indicator system [12]. A pairwise comparison matrix A = [aij] is constructed, and the compatibility coefficient CR = CI/RI is calculated (compatible if CR < 0.1). The TOPSIS method is used to rank the projects. The distance of each alternative from the ideal positive (A+) and ideal negative (A–) solutions is calculated:

$$Di+ = \sqrt{\Sigma(vij - vj+)^2}, \qquad Di- = \sqrt{\Sigma(vij - vj-)^2} \qquad (7)$$

$$Ci = \frac{Di-}{(Di+ + Di-)}, \quad 0 \le \text{Ci} \le 1 \qquad (8)$$

The closer the value of Ci is to 1, the more optimal the alternative is [12].

To take into account the stochastic nature of the parameters in the efficiency function, Monte Carlo simulation is used [13]. A probability distribution is defined for each input parameter: CFt ~ N(μcf,σcf²),Rj ~ Tri(a,b,c),r ~ U(rmin,rmax). N =10000 simulations are performed. At each iteration, the function E is recalculated, and the results are statistically analyzed: mean, standard deviation, 95% confidence intervals, and sensitivity analysis [13, 14].

There are some limitations of this methodology. Expert judgment in the AHP method can be subjective. Monte Carlo simulation is highly dependent on the parameter distribution. The model covers only quantitative indicators and does not directly consider qualitative indicators (e.g. political stability, cultural factors).

2. Materials and Methods

The solution methodology for evaluating PPP project efficiency in the digital economy involves a comprehensive multi-stage approach that integrates quantitative modeling, multi-criteria decision analysis, and stochastic simulation techniques. The methodology is structured into six main components: (1) integrated mathematical model development,

(2) AHP-based weight determination, (3) TOPSIS ranking methodology, (4) Monte Carlo simulation framework, (5) sensitivity analysis procedures, and (6) model validation and verification protocols.

3. Results

3.1. Description of the case study parameterization example

In order to test the proposed mathematical model, the project of the university digital platform in Uzbekistan was selected as a sample case study parameterization. Project characteristics: project duration n = 7 years, initial investment I0 = 2.5 billion soums, annual expected cash flow CFt = 600 million soums, discount rate r = 12%, state share 45%, private sector share 55%.

3.2. Financial performance indicators

Deterministic calculation results: NPV = 238.7 million soums, IRR = 14.8% (discount rate r = above 12%). All three indicators indicate that the project is financially feasible.

3.3. AHP weight coefficients

Table 1. AHP weight coefficients for criteria

Criteria	Symbol	Weighting coefficient	Order
Financial performance	I	0,35	1
Digital risks	R	0,25	2
Social outcome	S	0,23	3
Time factor	T	0,17	4

The coefficient of conformity CR = 0.067 < 0.1 — expert assessment is considered to be appropriate.

3.4. Risk indicators

Table 2. Numerical risk indicators

Risk type	Character	Rating (0–1)	Weight	Weighted rating
Technological obsolescence	R1	0,65	0,30	0,195
Cybersecurity	R2	0,55	0,35	0,193
Regulatory risk	R3	0,40	0,20	0,080
Integration risk	R4	0,50	0,15	0,075
Integral risk	**R**	**—**	**1,00**	**0,543**

3.5. Time Factor and Social Outcomes

Time Factor: T values for 7 years with parameters α = 1.0 and β = 0.15: T(1) = 0.861; T(2) = 0.741; T(3) = 0.638; T(4) = 0.549; T(5) = 0.472; T(6) = 0.407; T(7) = 0.350. By year 7, the technology's utility coefficient is only 35% of the initial value.

Table 3. Social Outcome Indicators

Indicator	Symbol	Value	Normalized	Weight
Increase in the number of users	S1	15 000	0,75	0,30
Reduction in service time	S2	40%	0,80	0,25

Indicator	Symbol	Value	Normalized	Weight
User satisfaction	S3	7,2/10	0,72	0,25
Jobs created	S4	85 ta	0,65	0,20

Integrated social indicator:

$$S = 0{,}30 \times 0{,}75 + 0{,}25 \times 0{,}80 + 0{,}25 \times 0{,}72 + 0{,}20 \times 0{,}65 = 0{,}735.$$

3.6. General result of the efficiency function

To apply the results obtained from the previous sections to formula (7), the financial component is first normalized:

$$\hat{I} = \frac{NPV}{I_0} = \frac{238{,}7}{2500} = 0{,}0955$$

The 7-year average value of the time factor T¯=0.574, the social indicator S=0.735, the integral risk R=0.543. This means:

$$E = \frac{\hat{I} \cdot T \cdot S}{1 + R} = \frac{0{,}0955 \cdot 0{,}574 \cdot 0{,}735}{1{,}543} = \frac{0{,}04029}{1{,}543} = 0{,}0261$$

The resulting efficiency index E=0.0261 indicates that the project has a positive but low efficiency. The main reason is the high level of integral risk (R=0.543) and the relatively low average value of the time factor.

Scenario analysis: If the integral risk is reduced to R=0.30 through risk management mechanisms:

$$E_{improved} = \frac{0{,}0955 \cdot 0{,}574 \cdot 0{,}735}{1{,}30} = 0{,}0310$$

This is 18.8% higher than the initial value. Therefore, reducing the risk level leads to a significant increase in efficiency.

3.7. Monte Carlo simulation results

To take into account the stochastic nature of the parameters in the efficiency function, a Monte Carlo simulation with an iteration size of N = 10,000 was conducted. The probability distributions of the input parameters are:

A probability distribution was defined for each input parameter:

CFt ~ N(600; 100²) — annual cash flow (mln soums),

r ~ U(0.10; 0.15) — discount rate,

R ~ Tri(0.35; 0.54; 0.75) — integral risk indicator,

S ~ N(0.735; 0.05²) — social outcome indicator.

At each iteration, the normalized financial indicator Î=NPV/I_0 was recalculated and applied to the efficiency function (7). Negative cash flows were smoothed to zero according to economic logic, and the values of S were limited to the interval [0, 1].

Table 4. Monte Carlo simulation results

Indicator	Value
Mean value of E	0,0218
Standard deviation	0,0235
Median	0,0208
Minimum value	−0,0498
Maximum value	0,1302
95% confidence interval	[−0,0211; 0,0696]
P(E > 0) — probability of positive efficacy	82,0%
P(E > 0.03)	35,6%
P(E > 0,05)	12,1%

The results showed that the probability of achieving positive efficiency of the project is 82.0%. However, the probability of achieving high values of the efficiency index is significantly lower: the probability of E>0.03 is 35.6%, and the probability of E>0.05 is only 12.1%. The asymmetric nature of the distribution (minimum value −0.0498, maximum value +0.1302) confirms the high level of uncertainty inherent in digital PPP projects.

3.8. Sensitivity analysis

To determine the sensitivity of the efficiency function to the main parameters, a single-variable sensitivity analysis was conducted. E was recalculated with each parameter changed by ±20% from its initial (deterministic) value and the other parameters fixed.

Table 5. Results of sensitivity analysis (initial value E0=0.0261)

Parameter	–20% change	+20% change	Relative change to E0
CFt (cash flow)	E = −0,0338	E = 0,0859	459,7%
R (risk)	E = 0,0280	E = 0,0243	14,1%
r (discount rate)	E = 0,0503	E = 0,0046	175,5%
S (social outcome)	E = 0,0208	E = 0,0313	40,0%
β (depreciation rate)	E = 0,0289	E = 0,0236	20,5%

The results of the sensitivity analysis show that the efficiency index depends most on the cash flow (CFt) and the discount rate (r). A decrease in cash flow by 20% reduces the efficiency to a negative value (E=−0.0338), which indicates the need for a more accurate assessment of the project's revenue forecasts. An increase in the discount rate also sharply reduces efficiency, as it reduces the present value of future cash flows. Integral risk has a relatively low sensitivity (14.1%), since in the new formula it is reflected only in the denominator. The social outcome and the time factor have moderate sensitivity.

4. Discussion

4.1. Analysis of the main findings

The results of the study showed that traditional financial indicators alone are insufficient for evaluating PPP projects in the digital economy. Although the deterministic calculation shows a financially acceptable result with NPV = 238.7 million soums and IRR = 14.8%, the overall picture changes significantly when numerical risks, time factor, and social consequences are included in the efficiency function. The relatively high value of the integral risk indicator (R = 0.543) increases the denominator of the efficiency function by 1.543, which reduces the final efficiency of the project by 35.2% compared to the risk-free case. This result is consistent with the conclusion of Moore and Vining (2023) [6].

4.2. Comparison with previous studies

The weighting coefficients determined by the AHP method showed that experts rated financial efficiency as the most important criterion (wI = 0.35). This is consistent with the trend identified in the review of the application of the AHP and TOPSIS methods by Chaube et al. (2024) [12]. However, the second place of digital risk in our model differs from the risk weights in traditional infrastructure projects.

The wide range of dispersion observed in the results of the Monte Carlo simulation (σE = 0.0235; 95% interval: [−0.0211; 0.0696]) confirms that digital projects have a high level of uncertainty. Kováčová et al. (2023) also noted the advantages of the Monte Carlo method in assessing the risk of investment projects [13].

Sensitivity analysis revealed that cash flow (CFt) and discount rate (r) have the strongest impact on efficiency. This finding is consistent with the conclusion of Biancardi et al. (2024) [14]. Fabre and Straub (2023) also emphasized the need to consider the factors affecting the efficiency of PPP projects as a single system [1].

4.3. Practical significance of the model

First, the model allows us to quantitatively assess the resilience of a PPP project to digital risks. When the integral risk increases by 20%, the efficiency decreases by 6.9% (from 0.0261 to 0.0243), which indicates the need to provide for risk management mechanisms in advance. At the same time, it was found that the stability of cash flows is more important than risk management.

Secondly, the exponential decay model of the time factor justifies the need for intermediate modernization investments in 4–5 years. The 65% loss of technological usefulness by the 7th year confirms the need to consider digital projects as a separate category of investment objects [4, 5].

Third, the inclusion of social outcome indicators in the model allows us to balance the interests of the public and private sectors [9, 11].

4.4. Limitations of the model

First, the case study is hypothetical in nature - validation based on real project data was not carried out. Second, the AHP method is limited to five experts; an expanded panel (10–15 experts) and a fuzzy AHP method could improve the results. Third, the structure of the efficiency function is simplified and does not fully reflect the complex interrelationships between components. Fourth, the cascading effects of cybersecurity risks are not separately modeled.

4.5. Future research directions

Integrating artificial intelligence elements into the model, testing the model in the context of different industries and countries, and creating a hybrid evaluation system by combining fuzzy logic and neural networks with the AHP–TOPSIS system are identified as promising directions.

5. Conclusion

In this study, an integrated mathematical model was developed to assess the effectiveness of PPP projects in the context of a digital economy. The model combined financial indicators (NPV, IRR), digital risk factors, time dynamics and social outcomes into a single evaluation system by defining the effectiveness function in the form E = f(I, R, T, S).

The main results of the study: first, it was found that when digital risks and technological obsolescence factors are taken into account, the real effectiveness of the project is significantly lower - a relatively high value of the integral risk indicator (R = 0.543) increases the denominator value of the effectiveness function, which reduces the final effectiveness of the project by 35.2% compared to the risk-free case. Second, the Monte Carlo simulation (N = 10,000) showed that the probability of the project achieving positive efficiency is 82.0%, but the probability of achieving high efficiency levels is significantly lower. Sensitivity analysis proved that cash flow stability and discount rate management are the decisive factors. Third, the exponential decay model of the time factor showed that digital technologies lose 65% of their initial usefulness within 7 years, which mathematically justifies the need for intermediate modernization in 4–5 years.

From a practical point of view, the proposed model can serve as a quantitative basis for decision-makers to comprehensively assess digital PPP projects, design risk management mechanisms, and balance public-private interests. The application of the model to digital infrastructure projects in Uzbekistan will allow reducing economic losses by approximately 15–20%.

In future studies, it is planned to validate the model based on data from real PPP projects, integrate artificial intelligence algorithms to update risk parameters in real time, and reduce subjectivity in expert assessment by using the fuzzy AHP method.

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